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Turbulent friction factor for two-phase: Air-viscoelastic flows through a horizontal tube

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The fully developed two-phase turbulent isothermal Fanning friction factors for air–viscoelastic fluid flows through a horizontal tube were measured experimentally. The viscoelastic fluids studied were aqueous solutions of polyacrylamide (100, 200, and 500 ppm by weight). Over the range of the apparent Reynolds number (Re_a) from 10,000 to 100,000, the homogeneous model was found to be accurate enough for engineering prediction of turbulent friction factor for air–viscoelastic flows through horizontal tubes. A new correlation for the turbulent friction factor of air–viscoelastic plug flow is proposed.

Keywords: two-phase; friction factor; viscoelastic

Introduction

Applications of two-phase gas–liquid flows range from transfer systems, such as pumping, to those involving heat and/or mass transfer, such as nuclear reactors. The designer's key interests in adiabatic two-phase flows are in the void fraction, and frictional pressure drop, which depend on fluid properties, flowrates, and the size of equipment.

For a given two-phase flow, the gas–liquid interface can take several possible forms, based on channel geometry and orientation, resulting in various *flow regimes* (e.g., bubble flow). Most data in the literature on flow regimes in horizontal gas-Newtonian liquid flows deals with air and water (in tubes of 2–5 cm diameter).

To represent the effects of various system parameters (e.g., pipe size, fluid properties), several generalized flow regime maps have been developed (Al-Sheikh et al. 1970; Baker 1954; Bell et al. 1969; Collier 1972; Dsarasov et al. 1974; Fiori and Bergles 1966; Fisher and Yu 1975; Gardner 1977; Hewitt 1978; Kubie 1979; Kutateladze 1973; Mandhane et al. 1974; Reimann and John 1978; Sakaguchi et al. 1979; Schicht 1969; Scott 1963; Taitel and Dukler 1976; and Wallis and Dobson 1973) based on superficial velocities of the phases, and such dimensionless quantities as F, K, T, X, Φ, and Ψ. None of these generalized flow regime maps represents all the appropriate *transitions* in terms of a single set of parameters.

Several analytical models are available to describe the two-phase flows: the *homogeneous* model, *separated-flow* model, *drift-flux* model, etc. (Wallis 1969).

The homogeneous model (the simplest of all and semiempirical at best) assumes that velocities of the two phases are equal (i.e., slip ratio = 1). This reduces the homogeneous density and void fraction to the following:

$$\rho_{TP} = \varepsilon \rho_g + (1-\varepsilon)\rho_1 \tag{1}$$

$$\varepsilon = x/[x + (1-x)\rho_g/\rho_1] \tag{2}$$

The most widely used two-phase viscosity, μ_{TP}, is that of McAdams et al. (1942). The frictional pressure gradient is related to that for the gas phase or liquid phase flowing alone in the channel, in terms of frictional multipliers ϕ_g and ϕ_1. The single-phase pressure drops are given by:

$$(dp/dz)_g = 32 f_g \dot{m}_g^2 x^2/\rho_g \pi^2 D^5 \tag{3}$$

$$(dp/dz)_1 = 32 f_1 \dot{m}_1^2 (1-x)^2/\rho_1 \pi^2 D^5 \tag{4}$$

Separated flow model assumes that the two phases can have different velocities. The Lockhart-Martinelli (1949) correlation for frictional pressure drop is given in terms of pressure drop multipliers.

$$(\phi_1)^2 = 1 + C/X + 1/X^2 \tag{5}$$

$$(\phi_g)^2 = 1 + CX + X^2 \tag{6}$$

where C is a dimensionless parameter whose value depends on the nature of the phase-alone flows. The *drift–flux* model is the most sophisticated and takes into account the relative motion of the two phases.

Non-Newtonian fluids

Non-Newtonian fluids, such as food products and petrochemicals, are frequently encountered in these process industries. Knowledge of the flow behavior of these rheologically complex

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fluids is essential to an improved design of the equipment handling such fluids. An introduction to non-Newtonian fluids is provided by some excellent references (Bird et al. 1960; Middleman 1968; Skelland 1967; Wilkinson 1960).

The simplest mathematical model that describes the flow behavior of a non-Newtonian fluid is given by

$$\tau = K'\dot{\gamma}^n \tag{7}$$

Viscoelastic fluids are a special class of non-Newtonian fluids with elasticity, which, therefore, can sustain unequal normal stresses at the flow boundaries and interfaces. These fluids exhibit *drag reduction* (Cho and Hartnett 1982). As the polymer concentration is increased, friction factor monotonically decreases from that of the Newtonian solvent, at any given Reynolds number, up to a minimum value (*the friction factor asymptote*). With further increase in polymer concentration, the solution becomes more viscous and increasingly elastic, but the friction factor will be unaffected. This anomaly has to do with the definition of the apparent Reynolds number.

The minimum concentration of polymer needed to achieve the friction factor asymptote depends on the chemistry of the solvent used, the polymer itself, and the cross section of the flow passage. Kwack (1983) proposed empirical formulae for friction factor asymptote (as a function of Re_a) and turbulent friction factor (as a function of Ws, Re_a) for single-phase viscoelastic flows in straight tubes. The latter has very limited applicability because it holds only at two particular values of Re_a.

Experimental setup and procedure

The schematic of the flow loop is shown in Figure 1. The test section is an opaque (PVC) tube ($D = 0.025$ m; $L = 11$ m). Only plastic plumbing items were used in order to minimize chemical reaction with the test fluid.

The viscoelastic test fluid was prepared by dissolving Praestol® (polyacrylamide) in distilled water at room temperature (18°C). The test fluid concentrations were 100, 200, and 500 ppm (by weight).

Air was injected at about 15 diameters downstream of the test section entrance. The maximum flow rates of liquid and air were 4.2×10^{-3} m^3/s and 1.2×10^{-3} m^3/s, respectively. The flow rates and pressure drop were measured with calibrated instruments.

In the range of τ_w measured in the test section, a parabolic equation was fitted for the steady shear viscometric data ($\dot{\gamma}, \tau$) obtained using a Bohlin rheogoniometer, a Brookfield viscometer, and a capillary viscometer:

$$\ln \tau = a_0 + a_1 \ln \dot{\gamma} + a_2(\ln \dot{\gamma})^2 \tag{8}$$

τ_w was calculated from the measured Δp. Equation 8 was solved for $\dot{\gamma}$, for $\tau = \tau_w$. η is given by:

$$\eta = \tau_w/\dot{\gamma} \tag{9}$$

Then, μ_{TP} and Re_{aTP} were calculated (see Notation).

Notation

A_c	cross-sectional area of tube, m^2
b	constant, Equation 11
c	constant, Equation 11
d	constant, Equation 11
D	tube inside diameter, m
K'	consistency index of power-law fluid, kg s^{n-2}/m
L	distance between pressure taps, m
$\dot{m}$	mass flow rate, kg/s
$\dot{m}_t$	total mass flow rate $= \dot{m}_1 + \dot{m}_g$, kg/s
$\dot{Q}$	volume flow rate $= \dot{m}/\rho$, m^3/s
v	mean velocity $= \dot{Q}/A_c$, m/s
v_1	$\dot{Q}_1/A_c$, m/s
v_{TP}	$(\dot{m}_t)/(A_c\rho_{TP})$, m/s
z	axial distance, m
Δp	pressure drop, $2fLv^2\rho/D$, Pa

Dimensionless quantities

f	Fanning friction factor $= \tau_w/(\frac{1}{2}\rho v^2)$
f_N	turbulent Newtonian friction factor (Equation 11)
f_{TP}	$(dp/dz)D^5\pi^2\rho/(32\,\dot{m}^2)$
F	Froude number $= \{[\rho_g/(\rho_1 - \rho_g)]/(Dg)\}^{1/2} v_g$
K^2	$\mathrm{F}^2\,\mathrm{Re}_1$
n	power-law exponent
Re	Reynolds number $= \rho v D/\mu = 4\dot{m}/(\pi D \mu)$
Re_a	apparent Reynolds number $= \rho v D/\eta$
Re_g	$4\,\dot{m}_g/(\pi D \mu_g)$
Re_1	$4\,\dot{m}_1/(\pi D \mu_1)$
Re_{aTP}	$4\,\dot{m}_t/(\pi D \mu_{TP})$
T^2	$(dp/dz)_1/[(\rho_1 - \rho_g)g]$
x	mass quality $= \dot{m}_g/\dot{m}_t$
X^2	$\Delta p_1/\Delta p_g = (dp/dz)_1/(dp/dz)_g$
Ws	Weissenberg number $= \lambda v_1/D$

Greek

$\dot{\gamma}$	shear rate, s^{-1}
ε	volumetric quality $= \dot{Q}_g/(\dot{Q}_g + \dot{Q}_1)$
η	apparent viscosity $= \tau/\dot{\gamma}$ or $\tau_w/\dot{\gamma}$, (Pa.s)
λ	characteristic time of the viscoelastic fluid, s
μ	viscosity, (Pa.s)
μ_{TP}	$\{(x/\mu_g) + [(1-x)/\eta]\}^{-1}$, Pa.s
ρ	density of fluid, kg/m^3
ρ_{TP}	two-phase density, Equation 1
σ	surface tension, N/m
τ_w	wall shear stress $= \Delta p D/(4L)$, Pa
Φ	$[(\rho_g/\rho_a)(\rho_1/\rho_w)]^{1/2}$
ϕ_g^2	frictional multiplier $= \Delta p_{TP}/\Delta p_g$
ϕ_1^2	frictional multiplier $= \Delta p_{TP}/\Delta p_1$
Ψ	$(\sigma_w/\sigma)[(\mu_1/\mu_w)(\rho_w/\rho_1)]^{1/3}$

Subscripts

a	air; apparent
g	gas
l	liquid
m	measured
p	predicted
TP	two-phase
w	water; at the test section wall

The characteristic curves are shown in Figure 2. For each set of runs (with a fixed concentration of the polymer) a fresh batch of solution was prepared. The test fluid rheology was measured before and after each run to monitor the polymer degradation, and the variation was less than $\pm 2\%$.

Results and discussion

Based on the measured pressure gradients (not shown here), the present two-phase isothermal turbulent friction factors are in the fully developed region. The hydrodynamic entrance lengths for the present air–water and air–viscoelastic data are about 65 and 80 diameters, respectively. The reported entrance length for single-phase viscoelastic flows is 150 diameters (Cho and Hartnett 1982). No studies on hydrodynamic entrance length for turbulent two-phase gas–viscoelastic flows are reported in the literature.

Homogeneous model

The present experimental two-phase friction factors are shown as a function of the (Re_{aTP}) in Figure 3. From Figure 3 it can be seen that two-phase air–viscoelastic flows also exhibit *drag reduction*. The friction factor asymptote for two-phase air–viscoelastic flows is not reported in the literature. The present friction factor data, however, have not reached the asymptote for viscoelastic single-phase flows. The positive displacement pump used in this study posed a restriction on employing further higher concentration of the polymer. The viscosity of these polymer solutions dramatically increases with the concentration (see Figure 2).

The following correlation is proposed to predict the turbulent friction factor for two-phase air–viscoelastic and air–water flows as well as single-phase viscoelastic flows in straight tubes over a wide range of Re_a (6000 to 80,000).

$$f = f_N e^{-d\,Ws}(1 + b\,Ws)/Re_{aTP}^{c} \tag{10}$$

$$f_N = 0.0791\,Re^{-1/4} \tag{11}$$

The term f_N in Equation 10 is evaluated using Equation 11 at $Re = Re_a$ or Re_{aTP} depending on whether the flow is single-phase viscoelastic or two-phase. The empirical constants b, c, and d obtained by regression are 0.004, 0.0466, and -0.0477, respectively. The λs measured by the rheogoniometer are 0.001 s, 0.08 s, and 0.2 s for the 100, 200, and 500 wppm solutions, respectively. For Newtonian flows, Ws = 0. The Weissenberg number in Equation 10 is calculated using liquid phase superficial velocity v_l.

The present experimental friction factors are plotted in Figure 4 against the values predicted using Equation 10. The new correlation predicts the present data with an accuracy of $\pm 20\%$.

Despite its little *general* applicability to design problems, the *homogeneous* model is still the most widely accepted in design practice for predicting the two-phase friction factor.

Separated flow model

The two-phase pressure drop is sensitive to the flow regimes. Hence, the understanding and delineation of flow regime transitions is important. The dimensionless parameters K, F, and T for the present two-phase data are shown as a function of X in Figures 5–7. Flow visualization could not be made in the opaque test section used here. According to the two-phase generalized flow regime map for air-Newtonian liquids (Taitel and Dukler 1976), most of the present experimental two-phase data are in the *plug flow* region. The nature and prediction of the transition are still the subjects of active research. Also, assessment of the flow regime is somewhat subjective.

Lockhart-Martinelli correlation works better for the conditions $\mu_l/\mu_g > 10^3$ and $\dot{m}_g/A_c < 10^2$ kg/(m²/s), which the present data satisfy.

The frictional pressure drop multipliers ϕ_l and ϕ_g are shown as a function of Martinelli parameter (X) in Figure 8. The present two-phase data analysis revealed that the liquid phase flowing alone in the channel was turbulent, and the gas phase flowing alone was laminar. Under these conditions, the value of

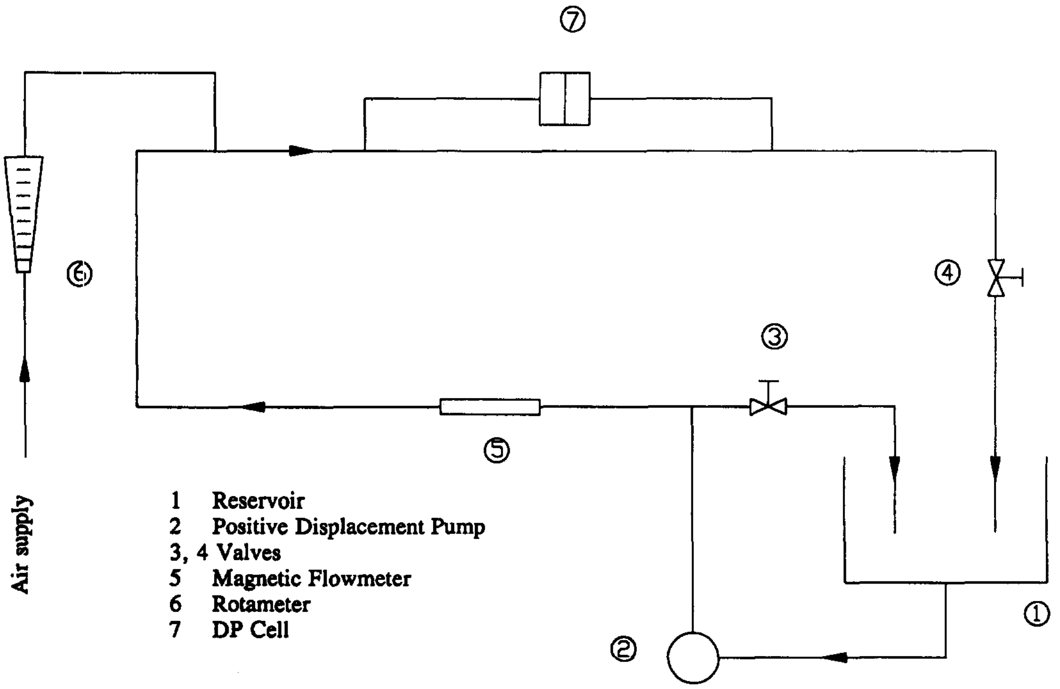

Figure 1 Schematic of the flow loop

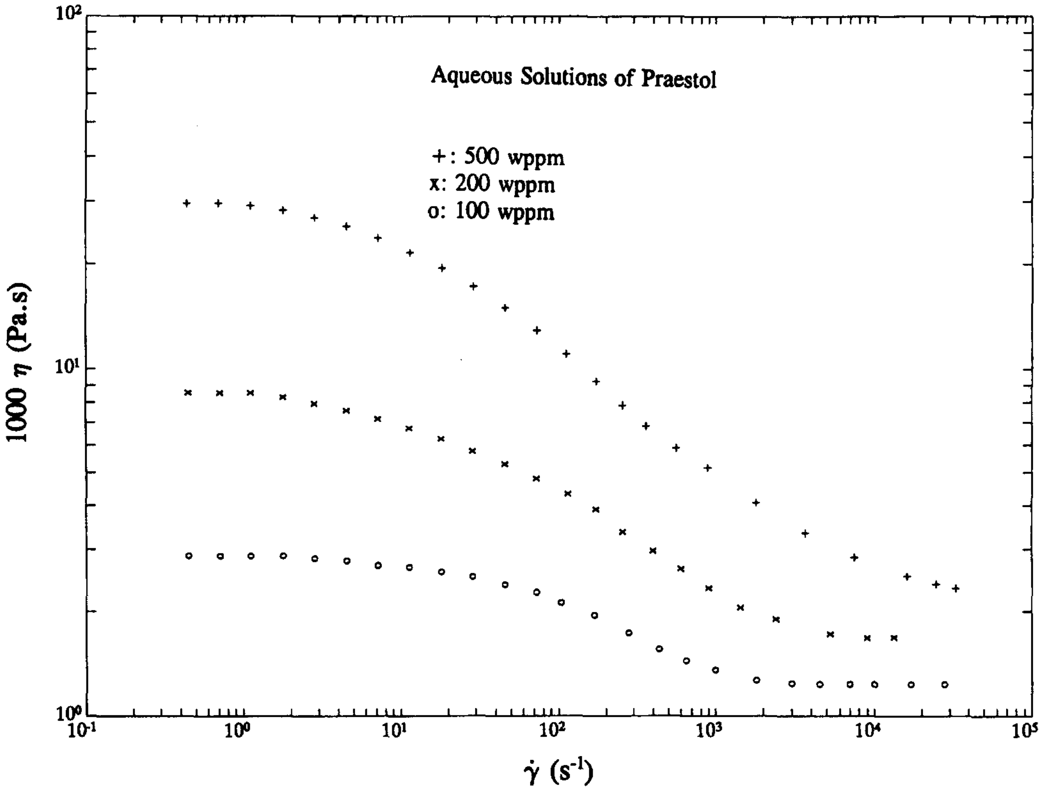

Figure 2 Characteristic curves of polymer solutions

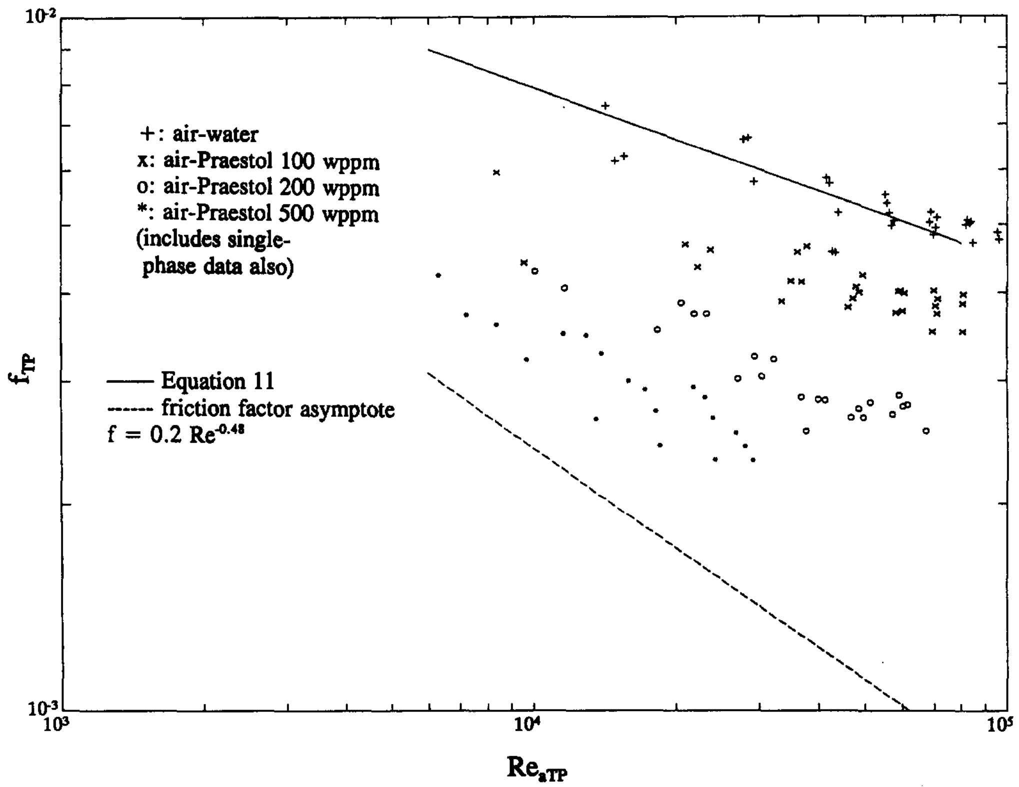

Figure 3 Turbulent friction factor versus Reynolds number for two-phase flows

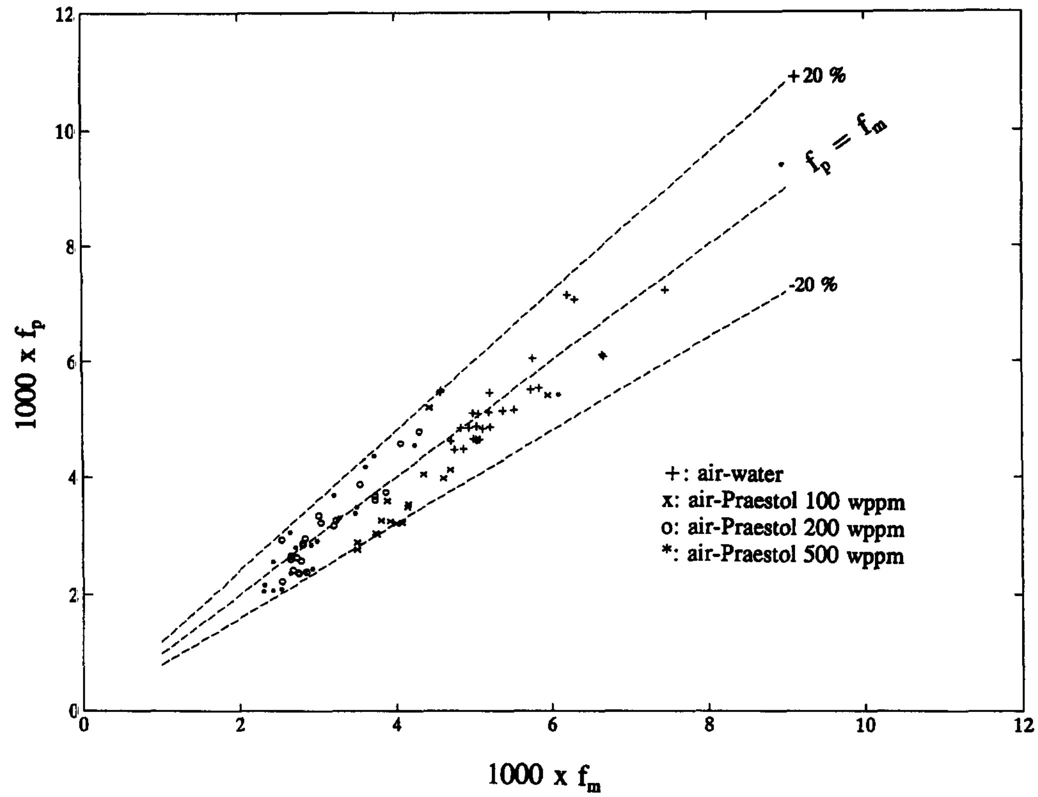

Figure 4 Predicted versus measured friction factors

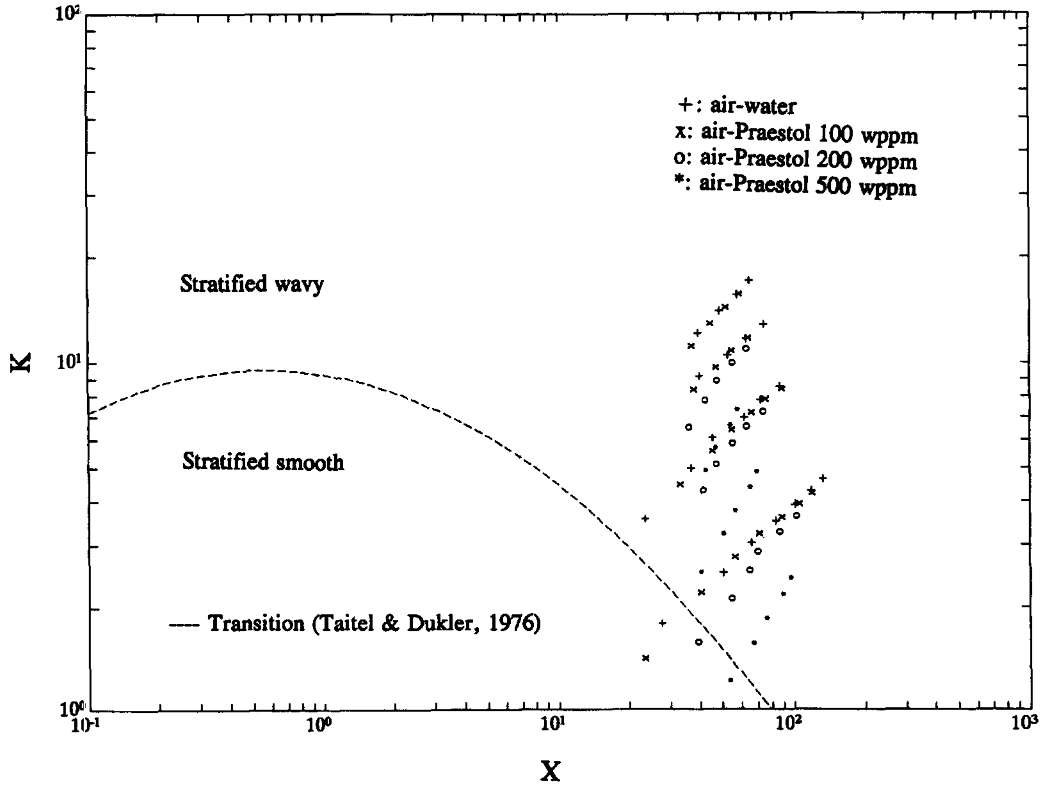

Figure 5 *K* versus *X*

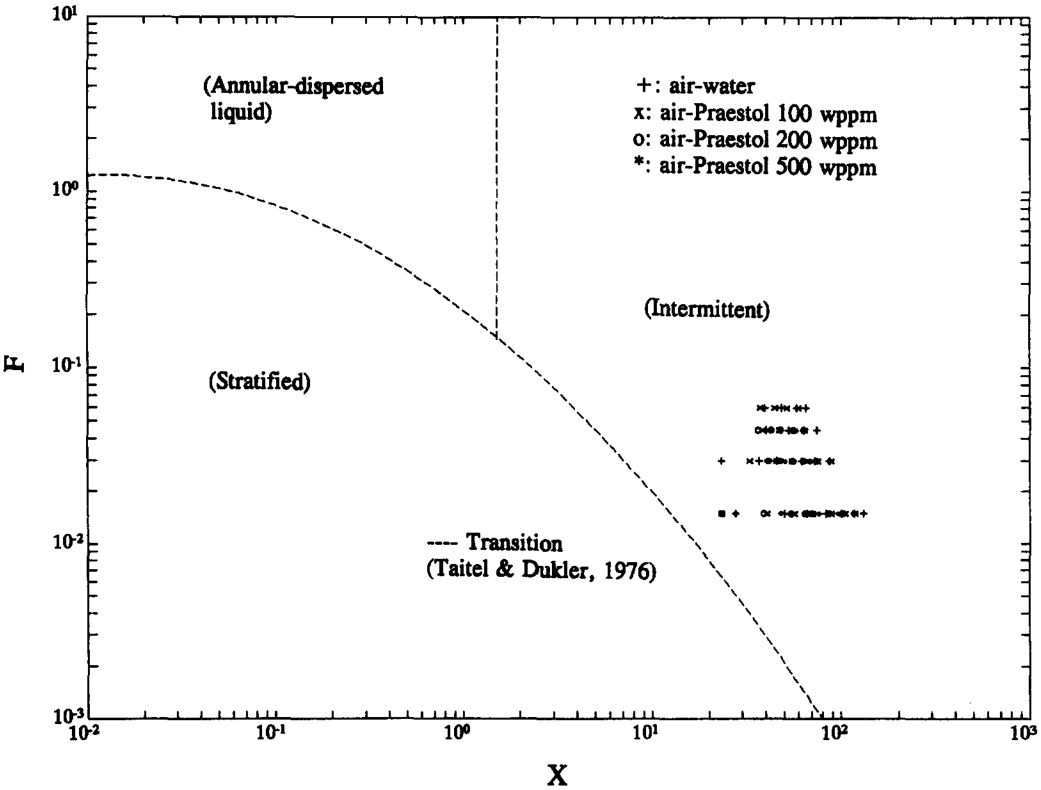

Figure 6 *F* versus *X*

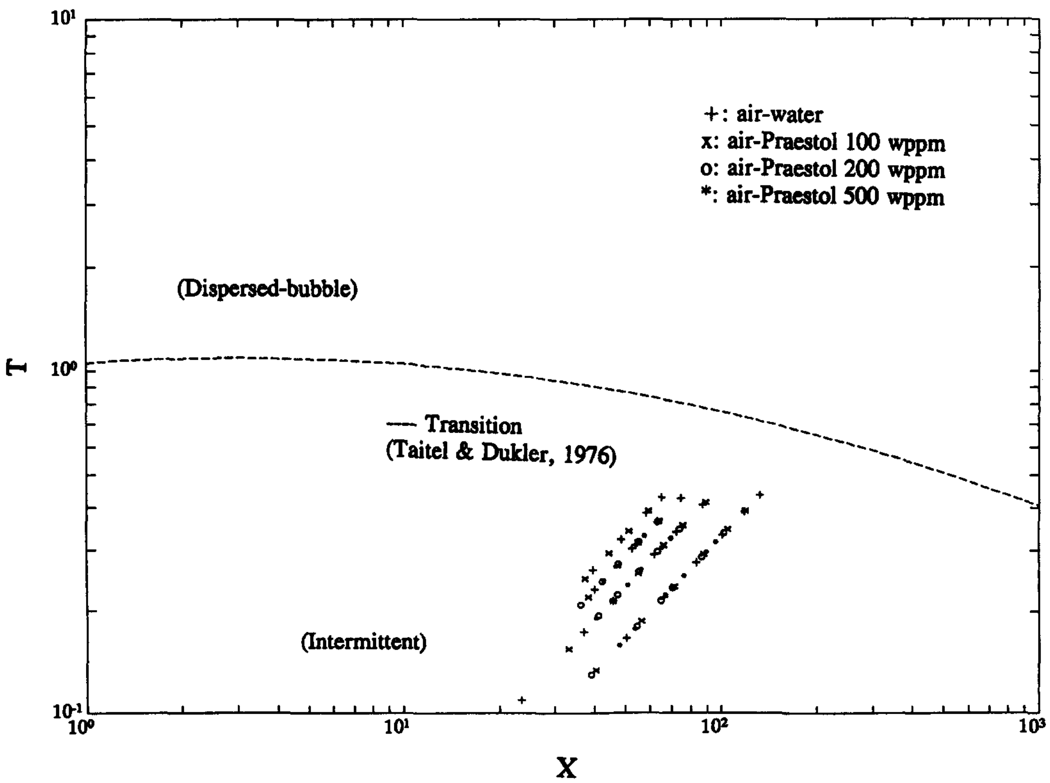

Figure 7 *T* versus *X*

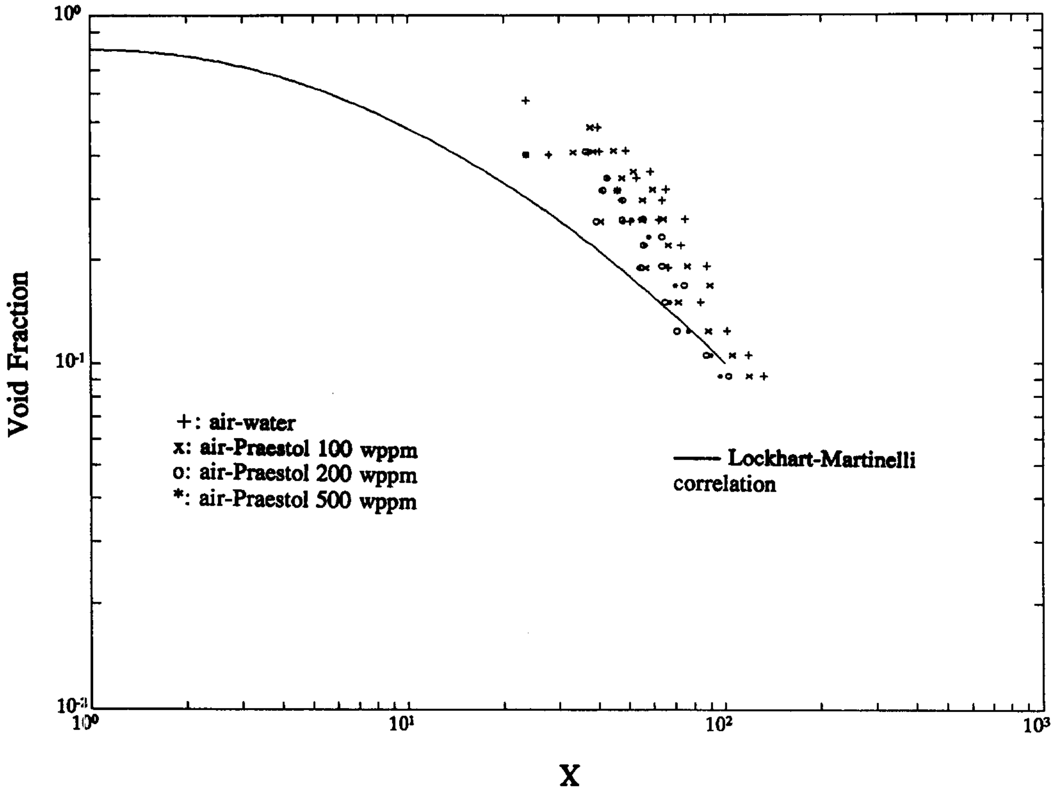

Figure 9 Void fraction versus X

C (for air-Newtonian liquid flows) in Equations 5 and 6 is equal to 10 (Chisholm 1967). The Lockhart-Martinelli multipliers (in Figure 8) reported for gas-Newtonian liquid flows predicted the present air–viscoelastic multipliers with reasonable accuracy.

The void fraction ε is shown as a function of X in Figure 9. The present experimental two-phase data are somewhat underpredicted by the Lockhart-Martinelli correlation for gas-Newtonian two-phase flows. Despite its deficiencies, most technical

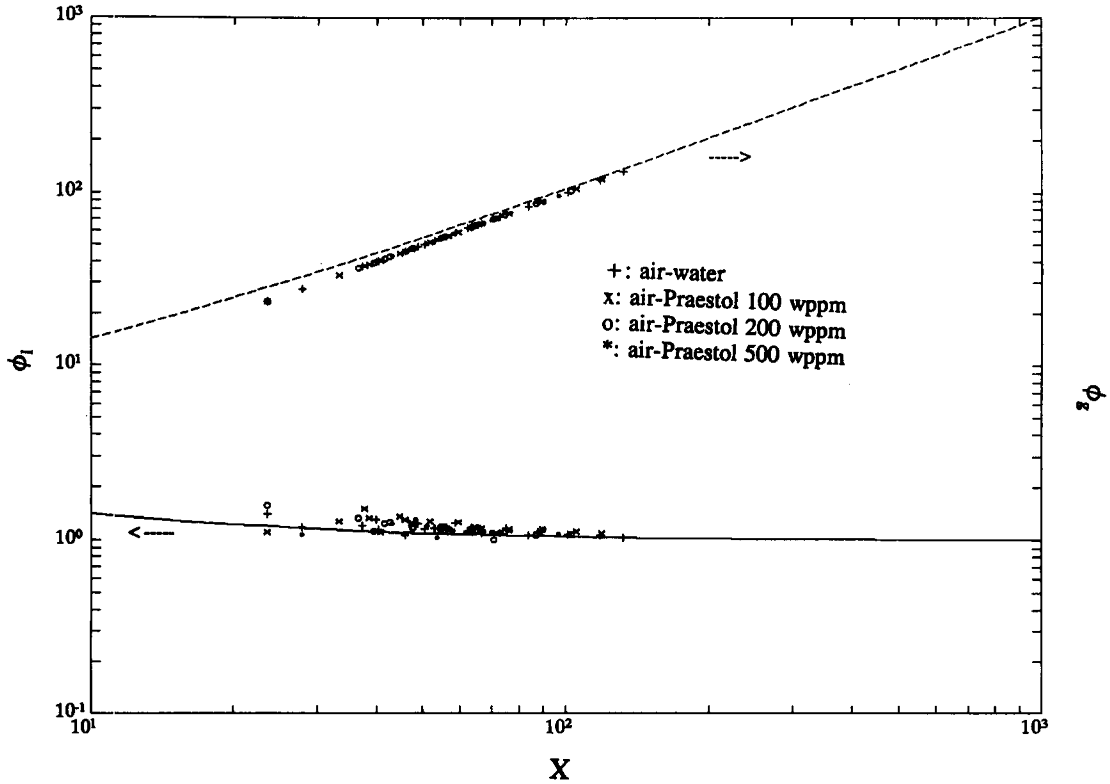

Figure 8 ϕ_l and ϕ_g versus X

calculations are still done using the method of Lockhart-Martinelli.

The error analysis (using the root-sum-square method) revealed that the uncertainties in the friction factors and Reynolds numbers in the present study are within ± 5 and $\pm 3\%$, respectively.

Conclusions

(1) The present data, based on the available generalized flow maps (Baker 1954; Taitel and Dukler 1976) are in the *plug flow* region.
(2) Equation 11 may be used to predict the turbulent two-phase friction factor for air–water flows.
(3) Air–viscoelastic flows exhibit *drag reduction* phenomena.
(4) Equation 10 can be used to predict the two-phase air–water, air–viscoelastic, and single-phase, viscoelastic, fully developed turbulent friction factor in straight tubes. More data are needed to verify the validity of this correlation for other polymers.
(5) At low pressures (up to 2 atmospheres), Lockhart-Martinelli correlation holds for predicting the two-phase pressure drop for air–water as well as air–viscoelastic turbulent flows in horizontal pipes. Lockhart-Martinelli correlation underestimates the void fraction for two-phase flows.

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